**FPT University**

**Course: Discrete mathematics**

**Course ID: MAD101**

**Student’s name: Group:**

**EXERCISES CHAPTER 4-5**

**PART I (3 MARKS)**

**Chapter 4. Induction & Recursion**

**Induction.**

1. Let P(n) be the statement that 12 + 22 +···+ n2 = n(n + 1)(2n + 1)/6 for the positive integer n.

a) What is the statement P(1)?

b) Show that P(1) is true, completing the basis step of the proof.

c) What is the inductive hypothesis?

d) What do you need to prove in the inductive step?

e) Complete the inductive step.

1. Prove that 3n<n! if n is an integer greater than 6.
2. Let P(n) be the statement that a postage of n cents can be formed using just 3-cent stamps and 5-cent stamps. The parts of this exercise outline a strong induction proof that P(n) is true for n ≥ 18.

a) Show statements P(18), P(19), P(20), and P(21) are true, completing the basis step of the proof.

b) What is the inductive hypothesis of the proof?

c) What do you need to prove in the inductive step?

d) Complete the inductive step for k ≥ 21.

**Recursion.**

1. Find f(2), f(3), and f(4) if f (n) is deﬁned recursively by f(1) = 1 and for n = 1, 2, 3,...
2. f(n) = f (n-1) + 5.
3. f(n) = 3f(n-1).
4. f(n) = 7
5. Find f(2), f(3), f(4), and f(5) if f is deﬁned recursively by f(0) =−1, f(1) = 2, and for n = 1, 2,...
6. f(n + 1) = f (n) + 3f(n − 1).
7. f(n + 1) = f(n − 1)/f (n).
8. f(n + 1) = f (n) − f(n − 1).
9. Give a recursive deﬁnition of the sequence {an}, n = 1, 2, 3,... if
10. an = -4n +3.
11. an = 3.(-5)n
12. an = n
13. an = 0 if n is even and an = 1 if n is odd
14. Let A be the set of bit strings deﬁned recursively by

λ ∈ A

if x ∈ A, 0x1 ∈ A?

(λ is the empty string)

List at least 5 elements in A.

1. Give a recursive definition with initial condition(s).
2. The set 0369*…*.
3. The set 1591317*…*.
4. The set of strings 1111111111111111*…*.

**Chapter 5. Counting**

**Sum rule and product rule**

1. An ofﬁce building contains 27 ﬂoors and has 37 ofﬁces on each ﬂoor. How many ofﬁces are in the building?
2. A multiple-choice test contains 10 questions. There are four possible answers for each question.
3. In how many ways can a student answer the questions on the test if the student answers every question?
4. In how many ways can a student answer the questions on the test if the student can leave answers blank?
5. There are four major auto routes from Boston to Detroit and six from Detroit to Los Angeles. How many major auto routes are there from Boston to Los Angeles via Detroit?
6. How many bit strings are there of length eight?
7. How many bit strings of length ten both begin and end with a 1?
8. How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?
9. How many different functions are there from a set with 7 elements to sets with the following numbers of elements?

a) 3 b) 5 c) 6 d) 7

1. How many one-to-one functions are there from a set with six elements to sets with the following number of elements?

a) 3 b) 7 d) 4

**Inclusion – Exclusion Principle**

1. How many bit strings of length seven either begin with two 0s or end with three 1s?
2. How many bit strings of length 10 either begin with three 0s or end with two 0s?
3. How many positive integers between 1000 and 9999 inclusive
4. are divisible by 7?
5. are divisible by 5 or 7?
6. are not divisible by either 5 or 7?
7. Suppose that f (n) = f (n/3) + 1 when n is a positive integer divisible by 3, and f(1) = 1. Find
8. f(3).
9. f(27).
10. f(729).
11. Suppose that f (n) = 2f (n/2) + 3when n is an even positive integer, and f(1) = 5. Find
12. f(2).
13. f(8).
14. f(64).
15. f(1024).
16. Give a big-O estimate for the function f(n) = 2f (n/3) if f is an increasing function.

**PART II (3 MARKS)**

1. Give a recursive deﬁnition of Sm(n), the sum of the integer m and the nonnegative integer n.
2. Give a recursive deﬁnition of Pm(n), the product of the integer m and the nonnegative integer n.
3. A vending machine dispensing books of stamps accepts, only $1 bills, $1 coins, and $2 bills. Let *an* denote the number of ways of depositing *n* dollars in the vending machine, where the order in which the coins and bills are deposited matters.

Find a recurrence relation for *an* and give the necessary initial condition(s).

Then find a5, number of ways of depositing *5* dollars.

1. Find a recurrence relation for the number of ternary strings of length n that contain two

consecutive 0s.

1. What are the initial conditions?
2. How many ternary strings of length six contain two consecutive 0s?

**THE END**